The T-violating Effective Chiral Lagrangian

Will Hockings

University of Arizona

Outline of Talk

1. Introduction
   a) QCD $\bar{\theta}$ term and chiral symmetry
   b) Eliminating spurious terms
   c) Other QCD sources of T violation

2. T violation from $\bar{\theta}$ term
   a) Pions only
   b) Pions $+$ nucleons

3. T violation from quark EDM and chromo-EDM

4. Conclusions/Outlook

QCD $\bar{\theta}$ term and Chiral Symmetry

QCD Lagrangian (two quark flavors)

$$\mathcal{L}_{QCD} = \bar{q} i \not{D} q + \ldots + \frac{\bar{\theta} g_s}{32\pi^2} \varepsilon_{\mu\nu\rho\sigma} Tr\left(G^{\mu\nu} G^{\rho\sigma}\right)$$

Perform chiral rotation on q, then $\bar{\theta}$ term becomes

$$\mathcal{L}_{\not{T},\bar{\theta}} = \bar{\theta} \bar{q} (m_* + \tilde{m} \tau_3) i \gamma_5 q$$

$$m_* = \frac{m_u m_d}{m_u + m_d} \qquad , \qquad \tilde{m} = f(m_u, m_d)$$

Problem: τ_3 part generates terms in low energy EFT that cause vacuum instability

One solution: impose vacuum stability at quark level $\Rightarrow$ No τ_3 term

Baluni, 1979

Alternate approach: At hadronic level, use field redefinitions to eliminate spurious terms

WH, van Kolck, in progress

Obtain low energy EFT using chiral symmetry
$SU_L(2) \times SU_R(2) \sim SO(4)$

- $\bar{q}i\gamma_5 q$ is 4th component of $SO(4)$ vector

$$P = (\bar{q}\boldsymbol{\tau}q, \bar{q}i\gamma_5 q)$$

- $\bar{q}i\gamma_5\tau_3 q$ is 3rd component of $SO(4)$ vector

$$S = (\bar{q}i\gamma_5\boldsymbol{\tau}q, \bar{q}q)$$

Other sources of T violation

quark chromo-EDM: $\frac{i}{2}\bar{q}\left(\breve{d}_s + \breve{d}_v \tau_3\right)\gamma_5 \sigma_{\mu\nu} G^{\mu\nu} q$

quark EDM: $\frac{i}{2}\bar{q}\left(d_s + d_v \tau_3\right)\gamma_5 \sigma_{\mu\nu} F^{\mu\nu} q$

and Weinberg operator, 4-quark interactions...

Question: What is most general T-violating effective chiral Lagrangian given various quark-level sources?

T violation from $\bar{\theta}$ term (pions only)

$$\mathcal{L}^{(0)}_{TC,\pi} = \frac{1}{2} D_\mu \boldsymbol{\pi} \cdot D^\mu \boldsymbol{\pi} + S_4[\boldsymbol{\pi}, 0]$$

$D_\mu = D^{-1}\partial_\mu$, $D = 1 + \boldsymbol{\pi}^2/4f_\pi^2$,

$$S_4[\boldsymbol{\pi}, 0] = 2m_\pi^2 f_\pi^2 (1 - \boldsymbol{\pi}^2/4f_\pi^2)/D$$

$$\mathcal{L}_{\not{T},\pi} = S_3[\boldsymbol{\pi}, 0]\Big(1 + \alpha_1 (D_\mu \boldsymbol{\pi})^2 + \alpha_2 S_4[\boldsymbol{\pi}, 0] + \ldots\Big)$$

$S_3[\boldsymbol{\pi}, 0] = \bar{h}_0 \pi_3 / D$, $\qquad \bar{h}_0 = \mathcal{O}(\bar{\theta}\tilde{m} M^3_{QCD}/f_\pi)$,

$\alpha_i = \mathcal{O}(1/f_\pi^2 M^2_{QCD})$

$\mathcal{L}_{\not{T},\pi}$ can be eliminated by redefi ning $\boldsymbol{\pi}$:

$$\pi_i \to \pi_i + D\Big(\sigma^{(2)}\pi_i + \sum_{a=0}^{\infty}\delta^{(2a)}\pi_i\Big)$$

$$\delta^{(2a)}\pi_i = \frac{D}{4f_\pi^2 m_\pi^2}\Big[D_\mu\pi_i D^\mu\boldsymbol{\pi}\cdot\delta^{(2a-2)}\boldsymbol{\pi} - (D_\mu\boldsymbol{\pi})^2\delta^{(2a-2)}\pi_i + \frac{D}{2(D-1)}\pi_i D_\mu\boldsymbol{\pi}\cdot D^\mu\delta^{(2a-2)}\boldsymbol{\pi}\Big]$$

$$\delta^{(0)}\pi_i = \bar{h}_0\delta_{i3}/(2m_\pi^2), \quad \sigma^{(2)}\pi_i = \delta^{(0)}\pi_i\{\alpha_1(D_\mu\boldsymbol{\pi})^2 + \alpha_2 S_4[\boldsymbol{\pi},0]\}$$

T violation from $\bar{\theta}$ term (pions+nucleons)

Leading order:

$$\mathcal{L}^{(0)}_{TC,\pi N} = \mathcal{L}^{(0)}_{TC,\pi} + \bar{N} i v \cdot D N - \frac{g_A}{f_\pi} \bar{N} S_\mu \boldsymbol{\tau} \cdot D^\mu \boldsymbol{\pi} N + \ldots$$

$D_\mu = \partial_\mu + i\boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times D_\mu \boldsymbol{\pi})/4 f_\pi^2$ for nucleon

$$\mathcal{L}^{(1)}_{\not{T},\pi N} = -\frac{\bar{g}_0}{D} \bar{N} \boldsymbol{\tau} \cdot \boldsymbol{\pi} N + \frac{\bar{g}_1}{D} \pi_3 \bar{N} N$$

$\bar{g}_0 = \mathcal{O}(m_* \bar{\theta}/f_\pi)$, $\quad \bar{g}_1 = \mathcal{O}(\tilde{m} \bar{\theta}/f_\pi)$

$\bar{g}_0$ term generated by isoscalar $\bar{\theta}$ term – gives nucleon EDM contribution

$\bar{g}_1$ term ($\sim S_3[\boldsymbol{\pi}, 0]S_4[0, N]$) along with terms induced on $\mathcal{L}^{(0)}_{TC,\pi N}$ by $\delta^{(0)}\pi_i$ can be eliminated:

$$\pi_i \to \pi_i + \ldots + \frac{D\bar{g}_1}{2m_\pi^2}\delta_{i3}\bar{N}N + D\bar{N}\epsilon^{(1)}\pi_i N$$

$$\epsilon^{(1)}\pi_i = \frac{\lambda D}{16 f_\pi^4 m_\pi^2}\Big[\varepsilon_{jk3} v_\mu \tau_j \Big(\pi_k D^\mu \pi_i - \frac{D}{2(D-1)}\pi_i D^\mu \pi_k\Big) - 4 g_A f_\pi D_\mu \pi_i S^\mu \tau_3\Big]$$

And so on... Field redefinition is perturbative, i.e. each term contributes extra power of $\sim m_\pi/M_{QCD}$

T violation from quark EDM & chromo-EDM

Under chiral symmetry:

quark chromo-EDM term same as $\bar{\theta}$ term

quark EDM term has four separate pieces

$$\sim S_3 \quad , \quad \sim P_4 \quad , \quad \sim S_3 F_{34} \quad , \quad \sim P_4 F_{34}$$

$F_{34} =$ 34-comp. of antisymmetric rank 2 $SO(4)$ tensor

Quark chromo-EDM gives same terms in low-energy EFT as $\bar{\theta}$ term

Quark EDM gives all previous terms plus new terms, including

$$\bar{g}_2 \pi_3 \bar{N} \tau_3 N$$

Can $\bar{g}_2$ term be eliminated? If so, this has consequences for neutron and deuteron EDMs:

Then one-nucleon physics is identical for above sources

$\Rightarrow$ Look to two-nucleon systems, i.e. deuteron to distinguish

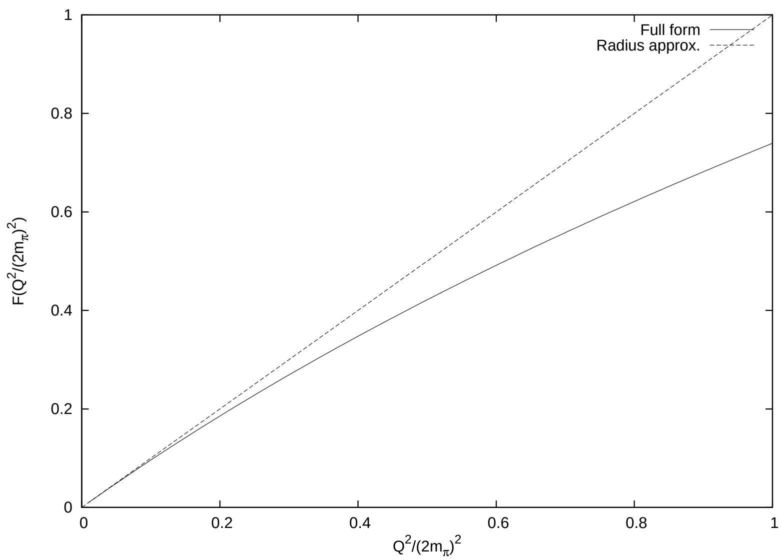
Application: nucleon electric dipole form factor

From $\bar\theta$ term in LO:

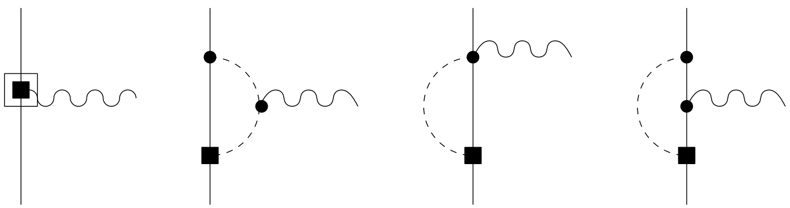
WH & van Kolck, 2005

Form factor from T-viol. EM current:

$$J^{\mu}_{ed}(q) = 2\left(F_D^{(0)}(-q^2) + F_D^{(1)}(-q^2)\tau_3\right)$$
$$\times\,(S^\mu v\cdot q - S\cdot q v^\mu)$$



Form factor contributions:



$$F_D^{(0)}(Q^2) = d_0$$

$$F_D^{(1)}(Q^2) = d_1 - \frac{eg_A\bar g_0}{12\pi^2 f_\pi}\,F\left(\frac{Q^2}{(2m_\pi)^2}\right)$$

$$F(x) = \frac{3}{2}\sqrt{1+\frac{1}{x}}\,\ln\left(\frac{\sqrt{1+1/x}+1}{\sqrt{1+1/x}-1}\right) - 3$$

Conclusions/Outlook

- The T-violating effective chiral Lagrangian consists of all terms consistent with the chiral symmetry properties of the quark-level sources of T violation

- Field redefi nitions provide a method of avoiding inclusion of terms in low-energy EFT which cause vacuum instability

- T-violating effective chiral Lagrangian allows accurate calculation of electric dipole moment/form factor of nucleon (c.f. WH, U. van Kolck, PLB **605**, 273 (2005) and WH, C.M. Maekawa, U. van Kolck, in progress) and deuteron